Department of Applied Mathematics
Preliminary Examination in Numerical Analysis
August 17, 2016, 10 am – 1 pm.

Submit solutions to four (and no more) of the following six problems. Show all your work, and justify all your answers. No calculators allowed.

1. Root finding / Nonlinear equations

Consider the scalar equation $F(x) = 0$. Assume L is a root of the equation.

a. Give the recursion for the Newton method for approximating a root.

b. Give conditions on $F(x)$ near L that guarantee convergence for x_0.

c. If, at the nodes $x = -h, 0, h$, one has function values y_{-h}, y_0, y_h

$s(x)$ that satisfies the data

x	-1	0	1	2
y	2	3	4	1

6.   Numerical PDE

Consider the parabolic equation

$$\frac{\partial u}{\partial t} = a\frac{\partial^2 u}{\partial x^2} + f$$

where a is a constant.

a.   Give the formula for the following finite difference approximations.
  (i)   Forward Euler: Centered differences in space, forward difference in time.
  (ii)   Backward Euler: Centered differences in space, backward difference in time.
  (iii)   Leapfrog: Centered difference in space and centered difference in time.

b.   What is the order of accuracy of each method?

c.   Use a von Neumann analysis (or any appropriate analysis) to determine the stability of each method.