S a i na Ex an i n Sh ck f a Regula i ed B u ine S em

By Gennady A. El, Mark A. Hoefer, and Michael Shearer

1. Introduction

(3) 1 $u_\pm$ $h_\pm$ 1 *RH locus*

$$u_\pm \quad h_\mp \left(\frac{2}{h \quad h} \right)^{1<2} \tag{4}$$

(2) (3) 1 1 1 (1),

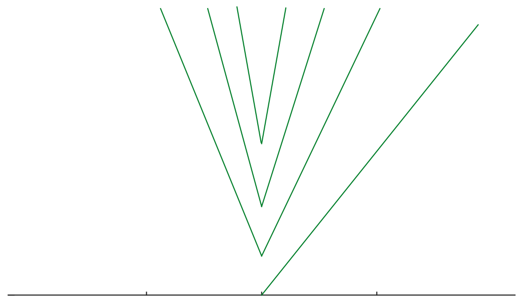
2. Expansion shock Riemann data

h_t (

1 1 , , ,
1 , ,
1 1 1
$h_\pm = \frac{1}{16}(r_\pm - s_\pm)^2$ (10),
1 (14) (15). 1
$s_\pm$, $r_\pm$ 1 H 1, 1 1
A H 1 1 (11
, (10)
1 1 ,
5.

3. BBM approximation and the structure of the expansion shock

1
(11) 1 (11 1
1 ,



4. Expansion shock for the Boussinesq equations

4.1. Inner solution: first -order approximation

$r^{(0)},\ r^{(1)},\ r^{(2)},\ s^{(0)},\quad s^{(2)}$ (1) $\to 0,\ \delta \to 0,\quad \to 0.$

1 1 1 1 r (15),

1 s $s^{(1)}$ 0,

1 ,

.

1 (21) (20),

$$\left(r^{(1)} \quad \cdots\right)$$

$$\frac{1}{4\delta}\left(3r^{(0)} \quad s^{(0)} \quad 3r^{(1)} \quad {}^{2}(3r^{(2)} \quad s^{(2)}) \quad \cdots\right)\left(r^{(1)} \quad {}^{2}r^{(2)} \quad \cdots\right)$$

$$\overline{6\delta^2}\left(r^{(1)} \quad {}^{2}\left(r^{(2)} \quad s^{(2)}\right) \quad \cdots\right) \tag{22}$$

$$\left(\quad {}^{2}s^{(2)} \quad \cdots\right) \quad \frac{1}{4\delta}\left(r^{(0)} \quad 3s^{(0)} \quad \cdots\right)\left({}^{2}s^{(2)} \quad \cdots\right)$$
$$\overline{6\delta^2}\left(r^{(1)} \quad \cdots\right). \tag{23}$$

B $\ll 1\delta,$ (22)

$$\left(\frac{}{\delta}\right): \quad \frac{1}{}$$

$$\frac{2a}{9a^2} \quad \frac{ff'}{f''} \quad K. \tag{30}$$

K / 0 1 . a f BB
5

$$a(\;) \quad \frac{A}{\frac{9}{2}AK \quad 1}. \quad f(\;) \quad B \left(\frac{B}{2K}\right. \quad . \tag{31}$$

A / 0 B / 0 1
1

$$B \quad 1. \quad K \quad \frac{1}{2}. \tag{32}$$

1 , 1 A,
, (31)

$$a(\;) \quad \frac{A}{\frac{9}{4}A \quad 1}. \quad f(\;) \quad (\;) \tag{33}$$

, 11 1

$$r^{(\,)}(\;.\;) \quad \frac{s^{(0)}}{3} \quad \frac{A}{\frac{9}{4}A \quad 1} \quad (\;) \quad (^{2}). \tag{34}$$

$$s^{(\,)}(\;.\;) \quad s^{(0)} \quad (^{2}) \tag{35}$$

1 $s^{(0)}$ A
, (34), (35) 0, $\to$, 1
, 1 1 (12)
1 :

$$r_\pm \quad \frac{s^{(0)}}{3} \pm A \quad (^{2}). \tag{36}$$

$s_\pm$

$> _t >> _x$ 0, $> \frac{1}{4}(s^{(0)}$ $3r)$ ((19)).
1 BB (18). ,
$>$
BB 1 5 .

E … 1 … 1 …

$$r^{( )}(\, . \,) \sim 1 \quad \left(\frac{1}{4} \quad \frac{( )}{1 \quad \frac{9}{4}} \right)$$

$$\frac{\ ^2}{3} \left(C \quad \frac{2 \quad 17 \quad ^2( ) \quad D \, ( ) \quad E \quad ( )}{16\left(1 \quad \frac{9}{4} \quad \right)^2} \right).$$

$$s^{( )}(\, . \,) \sim 3 \quad \frac{3}{4} \quad ^2 \left(C \quad \frac{3 \quad ^2( )}{16\left(1 \quad ^9\right.} \right.$$

$$\epsilon r^{(\;)} \quad \frac{1}{4}(3r^{(\;)} \quad s^{(\;)})r_X^{(\;)} \quad 0.$$

$$\epsilon s^{(\;)} \quad \frac{1}{4}(r^{(\;)} \quad 3s^{(\;)})s_X^{(\;)} \quad 0 \qquad (51)$$

1 1 , 1

$$s^{(\;)}(X. \;) \quad 3 \quad \frac{3}{4}\epsilon \quad \frac{1}{32}\epsilon^2 \quad \cdots.$$

$$r^{(\;)}(X. \;) \quad 1 \quad \epsilon\Big(\; \frac{1}{4} \quad r_1(X. \;) \quad \epsilon^2\Big(\; \frac{1}{96} \quad r_2(X. \;) \quad \cdots \qquad (52)$$

1 , (5) $s^{(\;)}$ r .
$r^{(\;)}$, 1 1 ϵ

(58). (50),

$$X\to 0^{\pm} \quad r_2(X. \quad) \quad F_2^{\pm}\frac{1}{\left(1 \quad \frac{9}{4} \quad\right)^2} \qquad \to\pm \quad r^{(2)}(\ . \ ) \quad \frac{1}{24\left(1 \quad \frac{9}{4} \quad\right)^2}. \tag{60}$$

F_2 F_2 1<24,

$$r_2(X. \quad) \quad \frac{1 \quad 3 \ X}{24\left(1 \quad \frac{9}{4} \quad\right)^2} \tag{61}$$

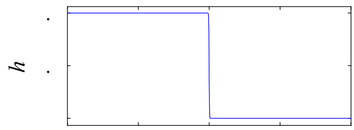
11

$$r^{(\ )}(X. \quad) \qquad 1 \quad \left(\frac{1}{4} \qquad \frac{X \quad 3X}{1 \quad \frac{9}{4}} \right)$$

$$\frac{\ ^2}{24}\left(\frac{1}{4} \quad 1 \quad 3 \right.$$

, 1

$r^{(\ )}(X. \ ) \quad 1$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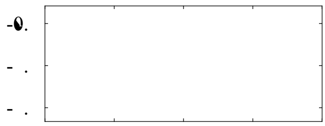

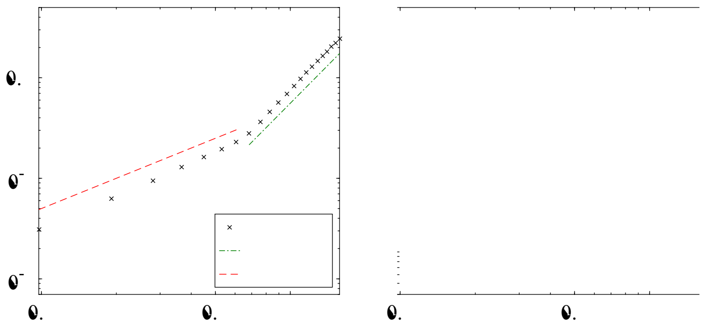
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6. Discussion

Acknowledgments

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Appendix

A 1 1 1 11 . L . L x_n L $2Ln<N$, n 0. ⋯ . N 1. 1 h u, 1 g h_x, $>$ u_x

$$g_t \quad (ug)_x \quad (h>)_x \quad 0.$$

$$> _t \quad (u>)_x \quad g_x \quad \frac{1}{3}>_{xxt} \quad 0 \tag{A1}$$

B L $h(\pm L . t)$ $h_\pm$ $u(\pm L . t)$ $u_\pm$ 10 9 g $>$ 1 . (A1) 1 , 1 1 1 , 1 ,

$$(ug)_{x\ n} \quad ik_n \ ug_{\ n} . \quad n \quad N<2. \cdots . N<2 \quad 1. \tag{A2}$$

, 1 1 - , k_n n $<L$ h 11 g

$$h(x_n . t) \quad h \quad {}^1 \ g(t)_{\ n} \quad \frac{1}{2L}(h \quad h\ )(x_n \quad L). \tag{A3}$$

$$g_n(t) \quad \begin{cases} \dfrac{2L}{N} \displaystyle\sum_{m \quad N<2}^{N<2 \quad 1} x_m g(x_m . t) & n \quad 0 \\ \dfrac{g_n(t)}{ik_n} & n \quad 0 \end{cases} \tag{A4}$$

n 0 (A4) 1 11 $\int_{L}^{L} xg(x.t)dx$, h g . A 1 1 u.

1 1 r

1 > xxt<3 (A1)

1 0.002 t 45. L 120 (.

2 3 1)

N 2^{14}. 1