This exam has 5 problems. Please start each new problem at the top of a new page in your blue book. Show all your work in your blue book and simplify your answers. Answers with missing or insufficient justification will receive no points. You are allowed one 8.5×11-in page of notes (TWO sided). You may NOT use a calculator, smartphone, smartwatch, the Internet or any other electronic device.

Problem 1(30 points) The following questions are not related:

(a) Find the value(s) of a and b where the function

$$I(a;b) = \int_a^b (-2x^2 + 10x - 12)\,dx$$

has a local maximum. Be sure to support your answer using Calculus 3 concepts.

(b) Consider the function

$$f(x;y;z) = x^2 y - z\cos(y)$$

Use a directional derivative to approximate how much f changes if one moves a distance of 0.1 from the point $(4;0;3)$ straight toward the origin.

(c) Arrange the following three double integrals in order from least to greatest and justify your reasoning

$$\int_0^2 \int_0^1 e^{x^2+y^2}\,dx\,dy; \qquad \int_0^2 \int_0^{2-\frac{y}{2}} e^{x^2+y^2}\,dx\,dy; \qquad \int_0^2 \int_0^{1-\frac{y}{2}} e^{x^2+y^2}\,dx\,dy$$

Problem 2(30 points)
Given the force vector field

$$F(x;y;z) = 2$$

Problem 4 (40 pts)
The following questions are not related:

(a) Suppose

$$4y\,dx + 7x\,dy = 13$$

where C is a simple, smooth curve oriented counterclockwise in the xy-plane that encloses the region R. Given only this information, is it possible to find the area of R? If so, find the area and justify your reasoning. If not, explain what additional information you'd need.

(b) Let dA give the area of a region R in the first quadrant of the xy-plane. (Note, this region is not related to the region R in part (a)). You are interested in the volume V generated by revolving R about the x-axis. If $g(x, y)\,dA$ is the integral that calculates the volume V, determine the integrand $g(x, y)$ and

$$\iint_R$$

Problem 5 (50 pts)
Consider the 3D solid object E that is bounded on the top by $z = 2$, on the bottom by $z = 0$ and on the sides by $x^2 + y^2 + z^2 = 8$
Let

$$G = y\,\mathbf{i} - x\,\mathbf{j} + 3z\,\mathbf{k}$$

(a) Sketch and shade a cross section of the object in the rz-plane. Label the axes and any intercepts.
(b) Calculate the volume of the object.
(c) Calculate the outward flux of G through the entire surface of the object E using an appropriate Calculus 3 theorem.
(d) Verify your answer to part (c) by separately calculating the flux through each surface (i.e. the top, the bottom and the side) and adding them together.

End Of Exam: Have a great summer!