APPM 2350—Exam 2

Wednesday Oct 20th, 6:30pm-8pm 2021

This exam has 4 problems. Please start each new problem at the top of a new page in your blue book. Show all your work in your blue book and simplify your answers. Answers with no justification will receive no

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$$U(x, y) = 1 + xy$$
, where $x > 0$ and $y > 0$

- (a) Give the equation of the level curve of U(x, y) that passes through the point (1, 2).
- (b) Sketch the level curve you found in part (a). Label the value of

- (a) Find and classify all critical points of T(x, y).
 (b) If you start at the location (x, y) = (1, 0) and you move along a straight path toward the point (

(a) The quadratic Taylor approximation of G(x, y) centered at the point (1, 0) is given by:

$$Q(x, y) = G(1, 0) + G_x(1, 0)(x - 1) + G_y(1, 0)y + \frac{1}{2}(G_{xx}(1, 0)(x - 1)^2 + 2G_{xy}(1, 0)(x - 1)y + G_{yy}(1, 0)y^2)$$

$$= -23 - 2(x - 1) - 5y + 2(x - 1)^2 + 4y^2 - 3(x - 1)y$$
Thus
$$G(3, -1) \quad Q(3, -1)$$

$$= -23 - 2(2) - 5(-1) + 2(2)^2 + 4(-1)^2 - 3(2)(-1)$$

$$= -4$$

(b) Taylor's error formula for a quadratic approximation is given by:

$$|E(x, y)| = \frac{M}{3!} x - 1 + y^{-3}$$

where M is an upper bound on the function's 3rd order mixed partial derivatives on the region. We are given

$$M = \frac{1}{4}, \qquad |y| \quad 1.5 \qquad \text{and} \ |E(x, y)| \quad \frac{1}{3}$$
$$\frac{1}{24} \quad x - 1 \ + \ 1.5 \quad ^{3} \quad \frac{1}{3}$$
$$= \qquad x - 1 \ + \ 1.5 \quad ^{3} \quad 8$$
$$= \qquad x - 1 \ + \ 1.5 \quad 2$$
$$= \qquad \boxed{x - 1 \ + \ 0.5}$$

Thus