1. (24 points) Determine whether each of the following series is absolutely convergent, conditionally convergent, or divergent. For this problem, and all subsequent problems, explain your work and name any test or theorem that you use.

   (a) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{k(k+1)}$

   (b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n \ln n}$

   (c) $\displaystyle\sum_{j=1}^{\infty} \frac{(3j)^j}{e^j (j+1)^j}$

   Solution:

   (a) The series satisfies the conditions of the Alternating Series Test and therefore converges. The question asks whether it converges conditionally or absolutely. We observe that the series

$$\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$$

   converges using the Limit Comparison Test. To see this, we note that $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2}$ converges since this is a p-series

We therefore consider the integral

$$\int_{2}^{\infty} \frac{1}{x\ln(x)}\mathrm{d}x$$

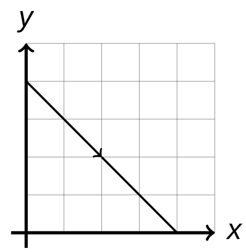
(d) To find the sum of the series, we begin with the geometric series given in the hint and note that $a = 1$ and the common ratio $r = (-1)(6x + 3)$. Thus,

$$f(x) = \sum_{k=0}^{\infty} (-1)^k (6x + 3)^k = \frac{1}{1 - (-1)(6x + 3)} = \frac{1}{6x + 4}$$

Solution:

(a)

which leads to



The graph appears to be a straight line starting at $(0,1)$ and ending at $(1,0)$; let's show that it is. Consider

$$x + y = \sin^2 t + \cos^2 t = 1 \Longrightarrow \boxed{y = 1 - x}$$

which is a straight line in standard Cartesian form.