APPM 1345
Exam 2
Spring 2024

Name	
Instructor Richard McNamara	**Section** 150

This exam is worth 100 points and has **4 problems**.

Make sure all of your work is written in the blank spaces provided. If your solutions do not fit, there is additional space at the end of the test. Be sure to **make a note** indicating the page number where the work is continued or it will **not** be graded.

Show all work and simplify your 5ua003 work is continued or it

1. (25 pts) Parts (a) and (b) are unrelated.

   (a) Find the average value f_{ave} of the function $f(x) = 9 \quad x^2$ on the interval $[0; 3]$, and find all values of c on $[0; 3]$ for which $f(c) = f_{\text{ave}}$.

Solution:

$$f_{ave} = \frac{1}{3}\int_0^3 (9 \quad x^2)\,dx = \frac{1}{3}\ 9x \quad \frac{x^3}{3}\ \Big|_0^3$$

$$= \frac{1}{3}(27 \quad 9) = \boxed{6}$$

$f\,(on\ the\ interval[0$

2.

4. (28 pts) Parts (a) and (b) are unrelated.

   (a) Consider the function $h(x) = \cos^2 x$ on the interval $I = [0; \pi/2]$.

      i. Determine the numerical value of the Riemann sum L_2 for $h(x)$ on I using **left** endpoints and 2 equally-sized subintervals. Fully simplify your answer.

      ii. Write an expression for the general Riemann sum L_n for $h(x)$ on I using **left** endpoints and n equally-sized subintervals. Express your answer using sigma notation.

Solution:

i. $\Delta x = \dfrac{b-a}{n} = \dfrac{\pi/2 - 0}{2} = \dfrac{\pi}{4}$

Since left endpoints are being used and the subintervals are of equal size, we have $x_0 = 0$ and $x_1 = \pi/4$.

$$\begin{aligned} L_2 &= [h(x_0) + h(x_1)]\,\Delta x \\ &= \left(\cos^2(0) + \cos^2(\pi/4)\right)\frac{\pi}{4} \\ &= \left[1^2 + \left(\frac{1}{\sqrt{2}}\right)^2\right]\frac{\pi}{4} \\ &= \frac{3}{2}\cdot\frac{\pi}{4} = \boxed{\frac{3\pi}{8}} \end{aligned}$$

ii. $\Delta x = \dfrac{b-a}{n} = \dfrac{\pi/2 - 0}{n} = \dfrac{\pi}{2n}$

Since left endpoints are being used and the subintervals are of equal size, we have

$$x_{i-1} = a + (i-1)\,\Delta x = \frac{(i-1)\pi}{2n}$$

$$\begin{aligned} L_n &= \sum_{i=1}^{n} h(x_{i-1})\,\Delta x \\ &= \sum_{i=1}^{n} \cos^2(x_{i-1})\,\frac{\pi}{2n} \\ &= \boxed{\frac{\pi}{2n}\sum_{i=1}^{n} \cos^2\left(\frac{(i-1)\pi}{2n}\right)} \end{aligned}$$

(b) Suppose the following expression is a Riemann sum for a continuous function $u(x)$ on the interval $[-1, 2]$:

$$R_n = \sum_{i=1}^{n} \left[\left(\frac{3i}{n} \right)^2 + 1 \right] \frac{3}{n}$$

Find the numerical value of $\int_{-1}^{2} u(x)\, dx$ by evaluating the appropriate limit of R_n. Do not use a Dominance